INPUT-OUTPUT REPRESENTATION OF THE ECONOMY

(REPRESENTAÇÃO INSUMO-PRODUTO DA ECONOMIA)

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. **ABSTRACT**

Based on W. Leontief's input-output economic theory, we propose a representation of the entire economy through a BPMN-grounded network in which each unit or economic agent receives inputs, demands a set of indirect costs, and produces a set of outputs. This representation enables monitoring the entire economy at varying levels of abstraction, as an economic unit may be composed of internal sub-units, thereby encapsulating them. This allows for visualizing the full network, calculating—via matrix algebra—wealth production by economic actors across the economy, optimizing the economic network (including through Artificial Intelligence), deriving diverse economic indicators, and generating visual representations for each economic unit. This fosters transparency and accountability for the economy as a whole. Finally, we demonstrate that an economy's wealth can be represented both by its financial assets and its material base, which would be proportional.

***K*eywords** Economy · Input-Output · BPMN · Graph · Representation

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Conflicts of Interest

The author of this manuscript declares no conflicts of interest regarding the publication of this article. The author has no financial or personal relationships that could influence or bias the work reported in this manuscript.

1 Introduction

According to Miller and Peter [2], input-output analysis is an analytical theory developed by Wassily Leontief in the 1930s that earned him the 1973 Nobel Prize in Economics. In its fundamental form, the input-output model consists of a system of linear equations describing the distribution of products in an economy.

The essential information in input-output analysis, according to this model, can be represented through an inter-industry transactions table, which shows the flow of products from each industrial sector (considered as producer) to every other sector (considered as consumer). In this representation, the rows of the table show the distribution of products from one producer to all economic sectors. The columns, in turn, represent the composition of inputs required by a particular sector to produce its outputs. Additional columns represent final markets, such as end consumers and the federal government, recording each sector's sales to these markets. There are also additional rows, called value added, representing other production inputs such as labor, capital depreciation, indirect industry taxes, and imports.

In this model, the transactions table can represent different aggregation levels, from national to state or even smaller regional levels. Moreover, the transaction units can be measured in physical quantities or monetary values. As can be

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easily observed, this model allows the use of linear equations and matrices to represent the interdependencies between economic sectors.

In the present work, we develop a similar approach where we represent the economy's production network through the interconnection of economic actors, which we call economic units. Each unit consumes a set of inputs and produces a set of outputs, forming a production network from original inputs to final products. This network can be represented at various aggregation levels, similar to a *Business Process Model and Notation - BPMN* representation (see ISO/IEC 19510:2013 [1]).

2 Results and Discussion

2.1 Theoretical Considerations

What I intend to represent in this model is the circulation and production of wealth in the economy. We know intuitively that economic wealth can be of two types: financial wealth and material-based wealth, composed of goods and services provided to society.

If we consider an exchange i between any two economic agents, we understand that their economic transactions can be of two types: one-time transactions u or recurring transactions r. In the first case, we represent the quantity of items exchanged by q_{iu} and the transaction price by p_{iu}. In the second case, we represent the quantity of items exchanged per unit time by q_{ir} and the transaction price per unit time by p_{ir}.

I maintain that there must be a value equivalence between the exchanged quantities of each product or service and their prices, i.e., $q_{iu} \sim p_{iu}$ and $q_{ir} \sim p_{ir}$. This occurs for two reasons: for fairness, as no producer will want to sell a highly valuable product or service at a meager price, nor will any buyer want to purchase a low-value product or service at an exorbitant price; and for market equilibrium, due to the law of supply and demand.

Thus, we can assume that $k_{ium} \leq p_{iu}/q_{iu} \leq k_{iuM}$, where k_{ium} is a minimum constant and k_{iuM} is a maximum constant, and that $k_{irm} \leq p_{ir}/q_{ir} \leq k_{irM}$, where k_{irm} is also a minimum constant and k_{irM} is another maximum constant.

Therefore, for n_u one-time transactions during time period T, we have: $\sum_{i=1}^{n_u}(k_{ium} \times q_{iu}) \leq \sum_{i=1}^{n_u}(p_{iu}) \leq \sum_{i=1}^{n_u}(k_{iuM} \times q_{iu})$. If we let $\overline{k_{um}} = \sum_{i=1}^{n_u}(k_{ium})/n)$ and $\overline{k_{uM}} = \sum_{i=1}^{n_u}(k_{iuM})/n)$, plus $Q_u = \sum_{i=1}^{n_u}(q_{iu})$ and $P_u = \sum_{i=1}^{n_u}(p_{iu})$, we obtain:

$$(\overline{k_{um}} \times Q_u) \leq P_u \leq (\overline{k_{uM}} \times Q_u) \tag{1}$$

Similarly, for n_r recurring transactions during time period T, we have: $\sum_{i=1}^{n_r}(k_{irm} \times q_{ir}) \leq \sum_{i=1}^{n_r}(p_{ir}) \leq \sum_{i=1}^{n_r}(k_{irM} \times q_{ir})$. If we let $\overline{k_{rm}} = \sum_{i=1}^{n_r}(k_{irm})/n)$ and $\overline{k_{rM}} = \sum_{i=1}^{n_r}(k_{irM})/n)$, plus $Q_r = \sum_{i=1}^{n_r}(q_{ir})$ and $P_r = \sum_{i=1}^{n_r}(p_{ir})$, we obtain:

$$(\overline{k_{rm}} \times Q_r) \leq P_r \leq (\overline{k_{rM}} \times Q_r) \tag{2}$$

If we let $k_m = \min(\overline{k_{um}}, \overline{k_{rm}})$, $k_M = \max(\overline{k_{uM}}, \overline{k_{rM}})$, $Q = (Q_u + Q_r)$, $P = (P_u + P_r)$ and $K = (k_m + k_M)/2$, we derive:

$$(k_m \times Q) \leq P \leq (k_M \times Q) \implies P = K \times Q \implies P \sim Q \tag{3}$$

In summary, in equilibrium, there must be an equivalence or proportionality between the total financial value of transactions during any given time period and the total quantity of goods or services circulating in the economy during that same period. This implies that both the total financial base of the economy and the material base of circulating goods and services can be used to represent the total wealth circulating in the economy.

2.2 Representation of the Economy

We can consider two fundamental types of actors or economic units: an activity, which characterizes recurring economic production, and a project, which characterizes unique economic production.

In the first case, the economic unit (activity) will consume a set of inputs $i_{a1} = (q_{ai1}, p_{ai1}), i_{a2} = (q_{ai2}, p_{ai2}), \ldots, i_{an} = (q_{ain}, p_{ain})$, characterized by pairs of unit quantities per unit of time and unit prices. Additionally, it will require, to operate, a set of indirect costs $c_{a1}, c_{a2}, \ldots, c_{ak}$ per unit of time and will produce a set of outputs $o_{ao1} = (q_{ao1}, p_{ao1}), (q_{ao2}, p_{ao2}), \ldots, (q_{aom}, p_{aom})$, also characterized by pairs of unit quantities per unit of time and unit prices.

In the second case, the economic unit (project) will consume a set of inputs $i_{p1} = (q_{pi1}, p_{pi1}), i_{p2} = (q_{pi2}, p_{pi2}), \ldots, i_{pn} = (q_{pin}, p_{pin})$, characterized by pairs of quantities and prices. Moreover, it will require, to operate, a set of indirect costs $c_{p1}, c_{p2}, \ldots, c_{pk}$ and will produce a set of outputs $o_{po1} = (q_{po1}, p_{po1}), (q_{po2}, p_{po2}), \ldots, (q_{pom}, p_{pom})$, also characterized by pairs of quantities and prices. In this case, the project will require, for its execution, a total time T.

For an activity x, if we represent, in column vectors, the input quantities per unit of time as $Q_{ai} = (q_{ai1}, q_{ai2}, \ldots, q_{ain})$, the unit prices of inputs as $P_{ai} = (p_{ai1}, p_{ai2}, \ldots, p_{ain})$, the output quantities per unit of time as $Q_{ao} = (q_{ao1}, q_{ao2}, \ldots, q_{aom})$, the unit prices of outputs as $P_{ao} = (p_{ao1}, p_{ao2}, \ldots, p_{aom})$, and the indirect costs per unit of time as $C_{ai} = (c_{ai1}, c_{ai2}, \ldots, c_{aik})$, with $U = (1, 1, \ldots, 1)$, then the profit or wealth produced per unit of time by this actor can be calculated as:

$$R_x = Q_{ao}^{\top} \cdot P_{ao} - Q_{ai}^{\top} \cdot P_{ai} - C^{\top} \cdot U \tag{4}$$

For a project y, if we represent, in column vectors, the input quantities as $Q_{pi} = (q_{pi1}, q_{pi2}, \ldots, q_{pin})$, the unit prices of inputs as $P_{pi} = (p_{pi1}, p_{pi2}, \ldots, p_{pin})$, the output quantities as $Q_{po} = (q_{po1}, q_{po2}, \ldots, q_{pom})$, the unit prices of outputs as $P_{po} = (p_{po1}, p_{po2}, \ldots, p_{pom})$, and the indirect costs as $C_{pi} = (c_{pi1}, c_{pi2}, \ldots, c_{pik})$, with $U = (1, 1, \ldots, 1)$, then the profit or wealth produced by this actor over the total period T can be calculated as:

$$R_y = Q_{po}^{\top} \cdot P_{po} - Q_{pi}^{\top} \cdot P_{pi} - C^{\top} \cdot U \tag{5}$$

Through this model, we can represent various types of actors in a standard economy: a manufacturing unit, with its inputs, indirect costs, and outputs produced per unit of time; a service unit, with its inputs, indirect costs, and services provided per unit of time; a commercial unit, with its inputs, indirect costs, and products sold per unit of time; the labor of an individual, whose inputs include wages received per unit of time, necessary indirect costs, and the goods or services provided per unit of time; a logistics unit, with the inputs or products received at the origin, the indirect costs of transportation, and the products delivered at the destination—whether this unit and its associated transportation are unique or recurring.

The economy as a whole can benefit from this representation of economic units by connecting all economic units in society, so that the outputs of each unit serve as inputs for other economic units, and the economy is represented as a large interconnected graph (or network). There would be original inputs, not produced by any actor, likely obtained from nature, and final consumers, closing the production cycle. Furthermore, individual actors could be aggregated into macro-economic units, such as a large corporation, with its internal economy represented by an internal production network. In this way, we can have economic units, aggregated or not, representing the entire economy—a country, a region, a city, the public sector of a nation, the private sector economy of a country, a company, public agency, or any institution, a household, an individual, or any other arbitrary aggregate.

It is important to note that this representation can be visual, in a computerized system allowing visualization at different levels of aggregation (through approximation—"zoom in"—and distancing—"zoom out"), for viewing at varying levels of abstraction. The economic units would be the responsibility of different individuals or institutions, but the information would be exchanged dynamically, at appropriate time intervals, between the computerized systems of the respective stakeholders (such as individuals, companies, and public agencies), enabling the entire economy to be represented and visualized dynamically.

As can be seen, this representation can be achieved using BPMN (see ISO/IEC 19510:2013 [1]) with varying degrees of abstraction. Additionally, since the results of each economic unit can be obtained through matrix algebra, the behavior of the entire economy could likewise be monitored through matrix calculations. In terms of data, we would need a list of all inputs and outputs in the economy, along with their units of measurement. Each actor would monitor the unit prices of these inputs/outputs both at entry/purchase and exit/production, as well as its indirect costs, to enable matrix-based calculations of results or wealth production.

The idea is to represent all economic actors in a graph or network, combinatorially, which can be grouped into macro-actors at different levels of abstraction or granularity, for individual or aggregated visualization. With all this dynamic information, where different stakeholders responsible for each economic unit exchange data dynamically, the entire economy can be monitored and visualized in real time. Through this representation, its data, and the use of matrix algebra, various types of analyses and optimizations could be performed, including the use of Artificial Intelligence. The wealth produced by each economic unit (or the entire economy) could be calculated in real time.

In terms of visualizing each economic unit, its inputs could be dynamically and visually represented in a vertical list, with information on name and unit of measurement, along with horizontal bars representing the quantities consumed and unit prices. The same could be done for outputs and indirect costs, providing a visual representation of the actor's dynamic behavior, including real-time wealth production calculations.

Another visualization, at a more aggregated level for this actor, would show a node (square or circle) with the following dynamic information: a list of inputs (quantities and unit prices) on the left, a list of outputs (quantities and unit prices) on the right, indirect costs at the bottom, and the wealth produced in real time at the top. This representation could be disaggregated in real time to show the internal network of connections between the various economic units within this actor, at increasing levels of detail (zoom in).

The following indicators could be calculated for each economic unit:

1. Net Wealth Creation Rate = $(Q_o^\top \cdot P_o - Q_i^\top \cdot P_i - C^\top \cdot U)/(Q_i^\top \cdot P_i + C^\top \cdot U)$;
2. Net Wealth Creation Rate (per input x) = $(Q_o^\top \cdot P_o - Q_i^\top \cdot P_i - C^\top \cdot U)/(Q_{xi} \cdot P_{xi})$;
3. Direct Costs = $Q_i^\top \cdot P_i$;
4. Indirect Costs = $C^\top \cdot U$;
5. Gross Wealth Produced = $Q_o^\top \cdot P_o$.

If we denote by I_i the column vector of taxes levied on each unit quantity of inputs, I_o the column vector of taxes levied on each unit quantity of outputs, and I_f the column vector of fixed taxes levied on the operation of actor x, then the result R_x, or wealth production of this actor, will be:

$$R_x = Q_o^\top \cdot (P_o - I_o) - Q_i^\top \cdot (P_i + I_i) - (C + I_f)^\top \cdot U \tag{6}$$

Thus, we can monitor the following additional indicators:

1. Taxes $I_x = Q_o^\top \cdot I_o + Q_i^\top \cdot I_i + I_f^\top \cdot U$;
2. Net Wealth After Taxes = $R_x - I_x$.

In terms of visual representation, we could have a vertical list of all taxes affecting the actor's operation, each accompanied by a horizontal bar representing its dynamic amount. We could also construct two dynamic graphs: a horizontal bar representing Net Wealth R_x for each tax rate t of a given tax; and another, showing two horizontal bars, the Net Wealth After Taxes and the total Taxes for this actor, for a configuration of dynamically adjustable rates for all taxes affecting the actor. Through these visual representations, local simulations could be performed to determine the maximum tax revenue or maximum post-tax wealth by varying the tax rates applicable to this actor.

This economic representation offers several advantages. Beyond the benefits related to control provided by dynamic data acquisition and visualization at different aggregation levels, there would be high transparency in the operation of each economic unit in the economy, low potential for tax evasion (since dynamic data could be stored for future audits), significant opportunities for optimization of the economy as a whole or for individual economic units—including through Artificial Intelligence—and the possibility of simulating economic behavior by locally varying parameters such as tax rates, indirect costs, and input/output prices and quantities.

3 Final Considerations

This study is grounded in the intuition underlying W. Leontief's economic theory and proposes, in a similar fashion, a representation of economic units through a column matrix of inputs (quantities and unit prices), a column matrix of indirect costs, and a column matrix of outputs (quantities and unit prices). Based on this representation of economic units as nodes, the entire economy can be modeled as a connected graph—or BPMN (Business Process Model and Notation)—with varying degrees of abstraction, since each economic unit may contain internal processes.

This economic model, to be maintained dynamically with individuals or institutions responsible for updating the data of specific economic units in real time, enables the graphical visualization of the entire economy, the calculation of various economic indicators and charts for each unit, accountability of economic agents, local simulations of actor economies when certain parameters are adjusted, and reduced opportunities for tax evasion. A final advantage is the ability to compute the behavior of economic units and the economy as a whole through matrix algebra, as well as to perform optimizations, including those leveraging Artificial Intelligence.

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